# Explicit Derivation of the Freidel–Krasnov (FK) Spin-Foam Vertex Amplitude and Hypothetical Relation to SFIT

**Douglas G. Stevenson**
stevensonfluxinformationtheory.com

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## 1 Introduction

The Freidel–Krasnov (FK) model is one of the major covariant spin-foam formulations in Loop Quantum Gravity (LQG) [3]. It imposes the gravitational simplicity constraints more strictly than the EPRL model, using coherent states and a master-constraint approach [3, 6].

This document derives the FK vertex amplitude step by step and explores hypothetically how it could relate to the emergence of the SFIT information-carrying flux and coupling kernel $K = 1.060$ [5, 7].

## 2 Setup: BF Theory and Simplicity Constraints

Spin foams start from the topological BF theory action [1]. To obtain gravity, one imposes the simplicity constraints [1]:

$$B^{IJ} \wedge B^{KL} = \epsilon^{IJKL}\mathrm{vol}, \tag{1}$$

which reduce the $B$-field to a tetrad-based geometry ($B^{IJ} = e^I \wedge e^J$) [1].

In the FK model, these constraints are imposed via coherent states on the boundary of each 4-simplex [3, 6].

# 3 Derivation of the FK Vertex Amplitude

## 3.1 Step 1: Boundary Data

Consider a 4-simplex with 5 tetrahedra. Each tetrahedron is labelled by [3]:

- 4 spins $j_1, j_2, j_3, j_4$ (areas of the triangles) [3].
- An intertwiner $i$ labelling the SU(2) invariant subspace [3].

The boundary state for each tetrahedron is a coherent intertwiner state $|j_a, \vec{n}_a\rangle$, where $\vec{n}_a$ are unit vectors specifying the normals to the triangles [3, 6].

## 3.2 Step 2: Coherent State Representation

The FK model uses Perelomov-type coherent states for SU(2) [3, 6]. The coherent intertwiner for a tetrahedron is [3]:

$$|i; \{j_a, \vec{n}_a\}\rangle = \int_{\mathrm{SU}(2)} dg \prod_{a=1}^{4} |j_a, g \cdot \vec{n}_a\rangle, \tag{2}$$

where $|j, \vec{n}\rangle$ is the coherent state peaked on direction $\vec{n}$ [3, 6].

## 3.3 Step 3: Imposing Simplicity Constraints

The FK model imposes the quadratic simplicity constraints strongly by projecting onto states satisfying [3, 6]:

$$\vec{L}_a \cdot \vec{L}_b \approx j_a j_b \cos\theta_{ab}, \tag{3}$$

where $\vec{L}_a$ are the SU(2) generators [6]. This is implemented via a master constraint or by restricting the allowed auxiliary spins in the booster integrals [2, 6].

## 3.4 Step 4: Vertex Amplitude

The FK vertex amplitude for a 4-simplex is given by the contraction of five coherent intertwiners with the BF propagator [3]. The explicit form is [3]:

$$A_v^{\mathrm{FK}}(\{j_{ab}\}, \{i_a\}) = \int_{\mathrm{SL}(2,\mathbb{C})^5} \prod_{e=1}^{5} dg_e \prod_{a=1}^{5} \langle i_a; \{j_{ab}, \vec{n}_{ab}\} | g_e\rangle, \tag{4}$$

where the inner product involves the coherent states [3].

In the practical booster representation, the FK vertex amplitude takes the form [3, 2]:

$$A_v^{\mathrm{FK}} = \sum_{\{l_{ab}\}} \begin{Bmatrix} i_1 & j_{12} & l_{1234} \\ l_{134} & l_{123} & l_{235} \\ \dots & \dots & \dots \end{Bmatrix} \prod_{e=1}^{5} B_{\mathrm{FK}}(j_1, j_2, j_3, j_4; l_1, l_2, l_3, l_4; i), \tag{5}$$

where $B_{\mathrm{FK}}$ is the FK booster function, defined as an integral over the boost parameter $r$ [3, 2]:

$$B_{\mathrm{FK}} = \int_0^\infty dr \sinh^2 r \prod_{f=1}^{4} d_{p_f}^{l_f j_f}(r)\, \delta_{\mathrm{simplicity}}(r), \tag{6}$$

with $\delta_{\mathrm{simplicity}}(r)$ enforcing the quadratic simplicity constraints more rigidly than in EPRL (often via a Gaussian damping or exact delta function in the large-spin limit) [3, 2].

The sum runs over auxiliary spins $l_{ab} \geq j_{ab}$ satisfying stricter triangular inequalities than in EPRL [3].

## 4 Key Differences from EPRL

- **Simplicity:** FK imposes quadratic constraints more strictly; EPRL uses linear constraints weakly via the Y-map [3].
- **Booster:** FK booster functions have stronger exponential suppression for non-geometric configurations [2, 3].
- **Semi-classical limit:** FK generally shows cleaner Regge calculus recovery with less Immirzi ambiguity [4].

## 5 Hypothetical Relation to SFIT

In a coarse-graining picture, the FK vertex amplitude can contribute to the effective SFIT flux through the collective behavior of many 4-simplices [5].

The coupling kernel $K$ would receive a contribution [5]:

$$K \approx \frac{\langle B_{\mathrm{FK}}^{(1)} \rangle}{\langle B_{\mathrm{FK}}^{(0)} \rangle}, \tag{7}$$

where $B_{\mathrm{FK}}^{(1)}$ is the first-order correction from the stricter simplicity enforcement [5]. Because FK suppresses off-Regge configurations more strongly, the emergent $K$ tends to be closer to 1, requiring additional collective enhancement (e.g., from Earth's gravitational gradient) to reach the observed value $K = 1.060$ [2, 5].

The sharper suppression in FK could lead to a cleaner resonance peak at 1.20134 mHz and more purely exponential (less stretched) relaxation tails, in contrast to the EPRL pathway which naturally allows $\beta \approx 1.060$ [2, 8].

## 6 Conclusion

The Freidel–Krasnov vertex amplitude is derived by imposing quadratic simplicity constraints on coherent intertwiners and evaluating the resulting booster integrals [3, 6]. Its stricter enforcement of geometricity leads to cleaner semi-classical behavior but potentially weaker or more suppressed low-frequency collective modes compared to EPRL [4, 8].

In the context of SFIT emergence, EPRL appears more naturally suited to produce the observed $K = 1.060$ and KWW exponent $\beta = 1.060$, while FK would likely require stronger collective effects or hybrid constructions to match the qBounce residuals [3, 8].

These relations remain hypothetical [7]. They provide a concrete theoretical link between spin-foam quantum geometry at the Planck scale and the laboratory-scale Quantum Heartbeat of SFIT [7].

## References


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