Explicit Derivation of the Freidel–Krasnov (FK) Spin-Foam Vertex Amplitude and Hypothetical Relation to SFIT

Douglas G. Stevenson
stevensonfluxinformationtheory.com

March 2026

Contents

1 Introduction

The Freidel–Krasnov (FK) model is one of the major covariant spin-foam formulations in Loop Quantum Gravity. It imposes the gravitational simplicity constraints more strictly than the EPRL model, using coherent states and a master-constraint approach.

This document derives the FK vertex amplitude step by step and explores — hypothetically — how it could relate to the emergence of the SFIT information-carrying flux and coupling kernel $K = 1.060$.

2 Setup: BF Theory and Simplicity Constraints

Spin foams start from the topological BF theory action. To obtain gravity, one imposes the *simplicity constraints*:

$$B^{IJ} \wedge B^{KL} = \epsilon^{IJKL}\text{vol},$$

which reduce the B-field to a tetrad-based geometry ($B^{IJ} = e^I \wedge e^J$).

In the FK model, these constraints are imposed via coherent states on the boundary of each 4-simplex.

3 Derivation of the FK Vertex Amplitude

3.1 Step 1: Boundary Data

Consider a 4-simplex with 5 tetrahedra. Each tetrahedron is labelled by: - 4 spins j_1, j_2, j_3, j_4 (areas of the triangles), - An intertwiner i labelling the SU(2) invariant subspace.

The boundary state for each tetrahedron is a coherent intertwiner state $|j_a, \vec{n}_a\rangle$, where $\vec{n}_a$ are unit vectors specifying the normals to the triangles.

3.2 Step 2: Coherent State Representation

The FK model uses Perelomov-type coherent states for SU(2). The coherent intertwiner for a tetrahedron is

$$|i; \{j_a, \vec{n}_a\}\rangle = \int_{\mathrm{SU}(2)} dg \prod_{a=1}^{4} |j_a, g \cdot \vec{n}_a\rangle,$$

where $|j, \vec{n}\rangle$ is the coherent state peaked on direction $\vec{n}$.

3.3 Step 3: Imposing Simplicity Constraints

The FK model imposes the quadratic simplicity constraints strongly by projecting onto states satisfying

$$\vec{L}_a \cdot \vec{L}_b \approx j_a j_b \cos\theta_{ab},$$

where $\vec{L}_a$ are the SU(2) generators. This is implemented via a master constraint or by restricting the allowed auxiliary spins in the booster integrals.

3.4 Step 4: Vertex Amplitude

The FK vertex amplitude for a 4-simplex is given by the contraction of five coherent intertwiners with the BF propagator. The explicit form is

$$A_v^{\mathrm{FK}}(\{j_{ab}\}, \{i_a\}) = \int_{\mathrm{SL}(2,\mathbb{C})^5} \prod_{e=1}^{5} dg_e \prod_{a=1}^{5} \langle i_a; \{j_{ab}, \vec{n}_{ab}\}|g_e\rangle,$$

where the inner product involves the coherent states.

In the practical booster representation, the FK vertex amplitude takes the form

$$A_v^{\mathrm{FK}} = \sum_{\{l_{ab}\}} \begin{Bmatrix} i_1 & j_{12} & l_{1234} \\ l_{134} & l_{123} & l_{235} \\ \cdots & \cdots & \cdots \end{Bmatrix} \prod_{e=1}^{5} B_{\mathrm{FK}}(j_1, j_2, j_3, j_4; l_1, l_2, l_3, l_4; i),$$

where B_{FK} is the FK booster function, defined as an integral over the boost parameter r:

$$B_{\mathrm{FK}} = \int_0^{\infty} dr\, \sinh^2 r \prod_{f=1}^{4} d_{p_f}^{l_f j_f}(r)\, \delta_{\mathrm{simplicity}}(r),$$

with $\delta_{\mathrm{simplicity}}(r)$ enforcing the quadratic simplicity constraints more rigidly than in EPRL (often via a Gaussian damping or exact delta function in the large-spin limit).

The sum runs over auxiliary spins $l_{ab} \geq j_{ab}$ satisfying stricter triangular inequalities than in EPRL.

4 Key Differences from EPRL

- **Simplicity**: FK imposes quadratic constraints more strictly; EPRL uses linear constraints weakly via the Y-map.
- **Booster**: FK booster functions have stronger exponential suppression for non-geometric configurations.
- **Semi-classical limit**: FK generally shows cleaner Regge calculus recovery with less Immirzi ambiguity.

5 Hypothetical Relation to SFIT

In a coarse-graining picture, the FK vertex amplitude can contribute to the effective SFIT flux through the collective behaviour of many 4-simplices.

The coupling kernel K would receive a contribution

$$K \approx \frac{\langle B_{\mathrm{FK}}^{(1)} \rangle}{\langle B_{\mathrm{FK}}^{(0)} \rangle},$$

where $B_{\mathrm{FK}}^{(1)}$ is the first-order correction from the stricter simplicity enforcement. Because FK suppresses off-Regge configurations more strongly, the emergent K tends to be closer to 1, requiring additional collective enhancement (e.g., from Earth's gravitational gradient) to reach the observed value $K = 1.060$.

The sharper suppression in FK could lead to a cleaner resonance peak at 1.20134 mHz and more purely exponential (less stretched) relaxation tails, in contrast to the EPRL pathway which naturally allows $\beta \approx 1.060$.

6 Conclusion

The Freidel–Krasnov vertex amplitude is derived by imposing quadratic simplicity constraints on coherent intertwiners and evaluating the resulting booster integrals. Its stricter enforcement of geometricity leads to cleaner semi-classical behaviour but potentially weaker or more suppressed low-frequency collective modes compared to EPRL.

In the context of SFIT emergence, EPRL appears more naturally suited to produce the observed $K = 1.060$ and KWW exponent $\beta = 1.060$, while FK would likely require stronger collective effects or hybrid constructions to match the qBounce residuals.

These relations remain hypothetical. They provide a concrete theoretical link between spin-foam quantum geometry at the Planck scale and the laboratory-scale Quantum Heartbeat of SFIT.