Explicit Derivation of M2-Brane Charge Quantization in M-Theory

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1 Introduction

In M-theory, the M2-brane is the fundamental electrically charged object under the 3-form gauge potential C_3. Its charge is quantized in integer multiples due to the consistency of the worldvolume action and the Dirac quantization condition generalized to higher dimensions.

This document derives the M2-brane charge quantization condition step by step, starting from the worldvolume action and ending with the flux quantization rule that appears in 11D supergravity.

2 M2-Brane Worldvolume Action

The low-energy effective action for a single M2-brane is the sum of a Dirac-Born-Infeld (DBI) term and a Wess-Zumino (WZ) term:

$$S_{\text{M2}} = -T_2 \int d^3\xi \, \sqrt{-\det(\gamma_{ij})} + T_2 \int_{\text{worldvolume}} C_3,$$

where: - $T_2 = (2\pi)^{-2}\ell_{11}^{-3}$ is the M2-brane tension, - $\gamma_{ij} = g_{MN}\partial_i X^M \partial_j X^N$ is the induced metric on the 3-dimensional worldvolume, - $C_3 = \frac{1}{3!}C_{MNP}\, dX^M \wedge dX^N \wedge dX^P$ is the pull-back of the 3-form gauge potential, - ℓ_{11} is the 11D Planck length.

The Wess-Zumino term couples the M2-brane electrically to C_3.

3 Field Strength and Flux

The 4-form field strength is

$$F_4 = dC_3.$$

In the 11D supergravity background sourced by the M2-brane, the equations of motion are

$$d * F_4 = 0 \quad \text{(away from sources)}, \qquad dF_4 = 0.$$

The electric charge of the M2-brane is measured by the flux of the dual 7-form through a 7-sphere surrounding the brane:

$$Q_2 = \int_{S^7} *F_4.$$

Derivation of Quantization Condition Consider a closed 7-sphere S^7 that links the M2-brane worldvolume. The integral of the dual field strength must be consistent with the Dirac quantization condition for extended objects.

The Wess-Zumino term can be written as

$$\int_{\text{worldvolume}} C_3 = \int_{S^7} *F_4 \cdot \frac{1}{2\pi} \quad \text{(after Stokes' theorem)}.$$

For the quantum theory to be well-defined, the phase factor in the path integral must be single-valued under large gauge transformations of C_3:

$$C_3 \to C_3 + d\Lambda_2,$$

where Λ_2 is a 2-form. The change in the Wess-Zumino term is

$$\Delta S_{\text{WZ}} = T_2 \int_{S^3} \Lambda_2.$$

Requiring the phase $e^{iS_{\text{WZ}}}$ to be invariant (or differ by $2\pi i n$) for integer winding numbers leads to the quantization condition

$$T_2 \int_{S^7} *F_4 = 2\pi n, \quad n \in \mathbb{Z}.$$

Substituting the tension $T_2 = (2\pi)^{-2}\ell_{11}^{-3}$ gives

$$\int_{S^7} *F_4 = 2\pi n\, \ell_{11}^3.$$

Thus, the M2-brane charge (electric flux) is quantized in integer units of $2\pi\ell_{11}^3$.

4 Magnetic Dual: M5-Brane Quantization

The magnetic dual object is the M5-brane, with charge

$$Q_5 = \int_{S^4} F_4 = 2\pi m\, \ell_{11}^6, \quad m \in \mathbb{Z},$$

where the integral is over a 4-sphere linking the M5-brane.

This duality is consistent with the self-duality of the 4-form in 11D supergravity.

5 Connection to SFIT

M-theory quantizes M2-brane flux at the Planck scale through the Wess-Zumino coupling and Dirac consistency. SFIT describes an effective low-energy resonant information flux at $\nu_{\text{res}} = 1.20134\,\text{mHz}$ with coupling kernel $K = 1.060$.

A possible synthesis is that the quantized M2-brane flux provides the microscopic origin of the information-carrying flux in SFIT. When M2-branes (or their collective excitations) interact with a macroscopic gravitational field, they may produce the observed 1.20134 mHz modulation and KWW tails with $\beta = K = 1.060$.

The non-reciprocal metric correction $h_{0z}^{\text{SFIT}}(t)$ in SFIT could be the coarse-grained 4D signature of the back-reaction of quantized M2-brane flux.

The 11.42 Hz secondary mode may represent a nonlinear mixing product arising from the quantized flux dynamics at laboratory scales.

6 Conclusion

The M2-brane charge quantization condition is

$$\int_{S^7} *F_4 = 2\pi n\, \ell_{11}^3, \quad n \in \mathbb{Z}.$$

This follows directly from requiring single-valuedness of the worldvolume path integral under large gauge transformations of C_3. The magnetic dual for the M5-brane is

$$\int_{S^4} F_4 = 2\pi m\, \ell_{11}^6, \quad m \in \mathbb{Z}.$$

These quantized fluxes source the 4-form in 11D supergravity and are the fundamental non-perturbative degrees of freedom of M-theory.

SFIT may capture the effective resonant behavior of these quantized M-brane fluxes when observed at laboratory energies in ultra-cold neutron experiments.