Subregion Duality in Holographic Duality
Detailed Explanation and Comparison with SFIT

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## Contents



# 1 Introduction

Subregion duality (also called subregion-subregion duality or entanglement wedge reconstruction) is one of the most important recent developments in holographic duality. It extends the idea that the bulk gravity is dual to the boundary quantum field theory by stating that **local physics in a bulk subregion is encoded in the entanglement structure of a corresponding boundary subregion**.

This document explains subregion duality in detail and compares it with Stevenson-Flux Information Theory (SFIT).

# 2 Core Concepts of Subregion Duality

## 2.1 From Global to Local Duality

The original AdS/CFT correspondence is a global duality: the entire bulk gravitational theory is dual to the entire boundary CFT. Subregion duality asks a sharper question:

¿ Which part of the bulk is encoded in which part of the boundary?

The answer involves three key geometric objects:

1. **Boundary Subregion $A$**: A spatial region on the conformal boundary. 2. **Ryu-Takayanagi Surface $\gamma_A$**: The minimal surface in the bulk homologous to $A$, whose area gives the entanglement entropy $S_A$:

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N \ell_{\text{P}}^{d-2}}.$$

3. **Entanglement Wedge $W_A$**: The bulk region bounded by $A$, $\gamma_A$, and the portion of the conformal boundary connecting them. It is defined as the domain of dependence:

$$W_A = D\left(A \cup \gamma_A\right).$$

### 2.2 Statement of Subregion Duality

Subregion duality asserts that: - The bulk physics inside the entanglement wedge $W_A$ is completely encoded in the reduced density matrix $\rho_A$ of the boundary subregion $A$. - Operators localized inside $W_A$ can be reconstructed from operators acting only on $A$. - Information outside the entanglement wedge cannot be reliably reconstructed from $A$ alone.

This is a powerful generalization: it allows **local bulk reconstruction** from boundary entanglement data.

## 3 Mathematical Details

The entanglement wedge $W_A$ is the causal domain of dependence of the region between the boundary subregion $A$ and its Ryu-Takayanagi surface $\gamma_A$. Any bulk operator $\mathcal{O}$ whose support lies entirely inside $W_A$ can be represented by a boundary operator $\mathcal{O}_A$ acting only on $A$:

$$\langle\psi|\mathcal{O}|\psi\rangle_{\text{bulk}} = \text{Tr}(\rho_A \mathcal{O}_A).$$

This reconstruction is possible because the entanglement wedge is the maximal region whose causal structure is protected by the boundary subregion.

The boundary between reconstructible and non-reconstructible regions is precisely the Ryu-Takayanagi surface. This leads to the concept of **entanglement wedge cross-sections** and connections to quantum error correction in holography.

## 4 Comparison with SFIT

| **Aspect** | **Subregion Duality (Holography)** | |
|---|---|---|
| Core Idea | Bulk subregion physics encoded in boundary entanglement | Gravity as dynam |
| Key Geometric Object | Entanglement wedge $W_A = D(A \cup \gamma_A)$ | Non-reciprocal |
| Information Flow | From boundary entanglement to bulk geometry | From gravitational flu |
| Scale | Holographic / Planck scale | Laboratory |
| Non-locality | Geometric via entanglement wedges | Directional |
| Testability | Indirect (requires strong holography) | Direct: qBounce (1 |
| Memory Effects | Encoded in wedge reconstruction | Manifest as KWV |

Table 1: Comparison of Subregion Duality and SFIT

### 4.1 Key Differences

- **Holographic Subregion Duality**: Information flows from boundary entanglement to bulk geometry. The entanglement wedge defines what part of the bulk is "visible" from a given boundary subregion.
- **SFIT**: Information is carried by an active gravitational flux. The flux at 1.20134 mHz directly modifies the metric tensor and induces phase-space skew in quantum systems. The KWW relaxation tails reflect the memory kernel of this flux.

### 4.2 Possible Complementary Relationship

Subregion duality and SFIT may be complementary. Holographic subregion duality provides the deep microscopic encoding where bulk physics is reconstructed from boundary entanglement via entanglement wedges. SFIT could describe the **effective low-energy resonant behavior** of this information flow when coupled to a macroscopic gravitational field.

In this picture: - The 1.20134 mHz Quantum Heartbeat could be a collective resonant mode arising from entanglement wedge dynamics in the presence of Earth's gravitational gradient. - The coupling kernel $K = 1.060$ quantifies how efficiently information from boundary subregions is transferred into measurable gravitational flux effects. - The KWW tails with $\beta = K$ reflect the slow relaxation of entangled degrees of freedom across entanglement wedges.

Thus, subregion duality may supply the ultraviolet holographic encoding, while SFIT describes the mesoscopic, observable manifestation at laboratory energies.

## 5 Conclusion

Subregion duality is a powerful extension of holographic principles, stating that bulk physics inside the entanglement wedge is encoded in the reduced density matrix of the corresponding boundary subregion. Together with the Ryu-Takayanagi formula and Complexity=Volume, it provides a geometric language for quantum information in holography.

SFIT offers a different but potentially complementary perspective: gravity as an active information-carrying flux that directly modifies the metric at laboratory scales. While holographic subregion duality geometrizes information flow from boundary to bulk, SFIT treats information as a dynamical flux that induces measurable effects in quantum systems.

Future ultra-cold neutron experiments (GRANIT) have the potential to test SFIT's predictions and indirectly illuminate aspects of holographic subregion duality at accessible energies.