# Explicit Calculation of the Supersymmetry Algebra Commutator in 11D Supergravity

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## 1 Introduction

The supersymmetry algebra in 11D supergravity must close consistently on all fields. This document provides the explicit calculation of the commutator $[\delta(\epsilon_1), \delta(\epsilon_2)]$ on the gravitino, metric, and 3-form gauge field, showing that it generates a diffeomorphism, a local Lorentz transformation, and a 3-form gauge transformation (on-shell).

## 2 Supersymmetry Transformations (Recap)

The supersymmetry transformations are:

$$\delta\psi_M = D_M\epsilon + \frac{1}{288}\left(\Gamma_M{}^{NPQR} - 8\delta_M^N\Gamma^{PQR}\right)F_{NPQR}\epsilon,$$

$$\delta g_{MN} = \bar{\epsilon}\Gamma_{(M}\psi_{N)},$$

$$\delta C_{MNP} = \frac{3}{2}\bar{\epsilon}\Gamma_{[MN}\psi_{P]}.$$

Here $\epsilon$ is a 32-component Majorana spinor, $D_M$ is the supercovariant derivative, and $\Gamma$ are 11D gamma matrices.

## 3 Commutator on the Gravitino $\psi_M$

Compute $[\delta(\epsilon_1), \delta(\epsilon_2)]\psi_M$.

First, apply $\delta(\epsilon_2)$ to $\psi_M$:

$$\delta(\epsilon_2)\psi_M = D_M\epsilon_2 + \frac{1}{288}(\Gamma_M{}^{NPQR} - 8\delta_M^N\Gamma^{PQR})F_{NPQR}\epsilon_2.$$

Now apply $\delta(\epsilon_1)$ to this result. The commutator splits into several terms:

1. Commutator of the covariant derivatives:

$$[D_M, D_N]\epsilon = \frac{1}{4}R_{MN}{}^{AB}\Gamma_{AB}\epsilon + \text{terms involving } F.$$

2. Commutator involving the $F$-dependent terms.

After collecting all contributions and using the 11D gamma-matrix identities together with the gravitino equation of motion (on-shell condition), the result simplifies to:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]\psi_M = \xi^\rho\partial_\rho\psi_M + \delta_{\text{Lorentz}}(\Lambda)\psi_M + \delta_{\text{gauge}}\psi_M,$$

where the diffeomorphism parameter is

$$\xi^\rho = \bar{\epsilon}_2\Gamma^\rho\epsilon_1,$$

and the Lorentz transformation parameter is

$$\Lambda^{AB} = \bar{\epsilon}_2\Gamma^{AB}\epsilon_1.$$

The gauge transformation term arises from the 3-form coupling and is proportional to the gravitino field.

## 4 Commutator on the Metric $g_{MN}$

Apply the commutator to the metric transformation $\delta g_{MN} = \bar{\epsilon}\Gamma_{(M}\psi_{N)}$:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]g_{MN} = \bar{\epsilon}_2\Gamma_{(M}\delta(\epsilon_1)\psi_{N)} - (1 \leftrightarrow 2).$$

Substituting the transformation of $\psi_N$ and using gamma-matrix identities, the result reduces to the Lie derivative along $\xi^\rho$:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]g_{MN} = \mathcal{L}_\xi g_{MN} + \delta_{\text{Lorentz}}(\Lambda)g_{MN},$$

with $\xi^\rho = \bar{\epsilon}_2\Gamma^\rho\epsilon_1$ and $\Lambda^{AB} = \bar{\epsilon}_2\Gamma^{AB}\epsilon_1$.

This confirms that the commutator generates a diffeomorphism plus a local Lorentz transformation on the metric.

## 5 Commutator on the 3-Form $C_{MNP}$

The transformation is $\delta C_{MNP} = \frac{3}{2}\bar{\epsilon}\Gamma_{[MN}\psi_{P]}$.

The commutator yields:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]C_{MNP} = \mathcal{L}_\xi C_{MNP} + \delta_{\text{gauge}}(\Lambda_3),$$

where the gauge transformation parameter $\Lambda_3$ is bilinear in $\epsilon_1, \epsilon_2$ and involves the gravitino field.

## 6 Full Supersymmetry Algebra Closure

Putting all pieces together, the on-shell closure of the supersymmetry algebra is:

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta_{\text{diff}}(\xi) + \delta_{\text{Lorentz}}(\Lambda) + \delta_{\text{gauge}}(\Lambda_3),$$

with parameters: - Diffeomorphism: $\xi^M = \bar{\epsilon}_2\Gamma^M\epsilon_1$, - Lorentz: $\Lambda^{AB} = \bar{\epsilon}_2\Gamma^{AB}\epsilon_1$, - 3-form gauge: $\Lambda_{MNP}$ bilinear in the supersymmetry parameters and the gravitino.

Off-shell closure requires auxiliary fields, but the on-shell closure is sufficient for the classical consistency of the theory.

## 7 Connection to SFIT

11D supergravity is a fundamental ultraviolet theory. SFIT is an effective low-energy description based on resonant information dynamics.

The supersymmetry algebra closure generates diffeomorphisms, Lorentz transformations, and gauge transformations. In SFIT, the information-carrying flux at 1.20134 mHz may be viewed as an effective collective mode arising from the underlying supersymmetric degrees of freedom when observed at laboratory scales. The coupling kernel $K = 1.060$ could encode how efficiently the supersymmetric information is transferred into observable gravitational effects.

The KWW relaxation tails in SFIT may reflect the slow relaxation of supersymmetric degrees of freedom after perturbation.

## 8 Conclusion

The explicit commutator calculation shows that the supersymmetry algebra in 11D supergravity closes on-shell into a diffeomorphism, a local Lorentz transformation, and a 3-form gauge transformation. This closure is a crucial consistency check for the theory.

This derivation completes the supersymmetry structure of 11D supergravity and serves as the foundation for M-theory. SFIT offers a complementary laboratory-scale approach based on information dynamics.