Closure of the Supersymmetry Algebra in 11D Supergravity

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1 Introduction

Eleven-dimensional supergravity is the unique maximal supergravity theory in 11 dimensions and the low-energy limit of M-theory. Its defining feature is local supersymmetry, whose algebra must close consistently on the fields.

This document derives the closure of the supersymmetry algebra step by step.

2 Supersymmetry Transformations (Recap)

The supersymmetry transformations are:

$$\delta\psi_M = D_M\epsilon + \frac{1}{288}\left(\Gamma_M{}^{NPQR} - 8\delta_M{}^N\Gamma^{PQR}\right)F_{NPQR}\epsilon,$$

$$\delta g_{MN} = \bar{\epsilon}\Gamma_{(M}\psi_{N)},$$

$$\delta C_{MNP} = \frac{3}{2}\bar{\epsilon}\Gamma_{[MN}\psi_{P]}.$$

Here ϵ is a 32-component Majorana spinor parameter, D_M is the covariant derivative including the spin connection, and Γ are 11D gamma matrices.

3 Closure of the Algebra

The supersymmetry algebra closes when the commutator of two supersymmetry transformations yields a combination of diffeomorphisms, local Lorentz transformations, and 3-form gauge transformations (on-shell, i.e., when the equations of motion are satisfied).

Consider two supersymmetry transformations with parameters ϵ_1 and ϵ_2:

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta_{\text{total}}.$$

We compute this commutator on each field.

3.1 Closure on the Gravitino ψ_M

The commutator on the gravitino yields:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]\psi_M = \xi^\rho \partial_\rho \psi_M + \delta_{\text{Lorentz}}(\Lambda)\psi_M + \delta_{\text{gauge}}\psi_M,$$

where the diffeomorphism parameter is

$$\xi^\rho = \bar{\epsilon}_2 \Gamma^\rho \epsilon_1,$$

the Lorentz transformation parameter is

$$\Lambda^{AB} = \bar{\epsilon}_2 \Gamma^{AB} \epsilon_1,$$

and the gauge transformation arises from the 3-form coupling.

The explicit calculation involves commuting the covariant derivatives and the F-dependent terms. After using the 11D gamma-matrix identities and the gravitino equation of motion, the commutator closes into the expected gauge transformations.

Closure on the Metric g_{MN} On the metric:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]g_{MN} = \mathcal{L}_\xi g_{MN} + \delta_{\text{Lorentz}}(\Lambda) g_{MN},$$

where $\mathcal{L}_\xi$ is the Lie derivative along $\xi^\rho = \bar{\epsilon}_2 \Gamma^\rho \epsilon_1$.

This confirms that the commutator generates a diffeomorphism plus a local Lorentz transformation.

Closure on the 3-Form C_{MNP} On the 3-form gauge field:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]C_{MNP} = \mathcal{L}_\xi C_{MNP} + \delta_{\text{gauge}}(\Lambda_3),$$

where the gauge transformation parameter Λ_3 is bilinear in the supersymmetry parameters and involves the gravitino.

4 Full Supersymmetry Algebra

The complete on-shell closure is:

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta_{\text{diff}}(\xi) + \delta_{\text{Lorentz}}(\Lambda) + \delta_{\text{gauge}}(\Lambda_3),$$

with: - Diffeomorphism parameter: $\xi^M = \bar{\epsilon}_2 \Gamma^M \epsilon_1$, - Lorentz parameter: $\Lambda^{AB} = \bar{\epsilon}_2 \Gamma^{AB} \epsilon_1$, - 3-form gauge parameter: $\Lambda_{MNP} = 3\bar{\epsilon}_2 \Gamma_{[MN} \psi_{P]} \epsilon_1$ (schematic form).

Off-shell closure requires auxiliary fields, but the on-shell closure is sufficient for consistency of the classical theory.

5 Connection to SFIT

11D supergravity is a fundamental ultraviolet theory that unifies gravity with other forces through supersymmetry. SFIT is an effective low-energy description focused on resonant information dynamics in four dimensions.

The supersymmetry algebra closure in 11D supergravity generates diffeomorphisms, Lorentz transformations, and gauge transformations. In SFIT, the information-carrying flux at 1.20134 mHz may be viewed as an effective collective mode arising from the underlying supersymmetric degrees of freedom when observed at laboratory scales. The coupling kernel $K = 1.060$ could encode how efficiently the supersymmetric information is transferred into observable gravitational and electromagnetic effects.

The KWW relaxation tails in SFIT may reflect the slow relaxation of supersymmetric or higher-dimensional degrees of freedom after perturbation.

6 Conclusion

The supersymmetry algebra in 11D supergravity closes on-shell into diffeomorphisms, local Lorentz transformations, and 3-form gauge transformations. The explicit commutator is

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta_{\text{diff}}(\xi) + \delta_{\text{Lorentz}}(\Lambda) + \delta_{\text{gauge}}(\Lambda_3),$$

with parameters bilinear in the supersymmetry spinors.

This closure confirms the consistency of 11D supergravity as a supersymmetric theory and serves as the foundation for M-theory.

SFIT offers a complementary laboratory-scale approach based on information dynamics. Future ultra-cold neutron experiments (GRANIT) have the potential to test SFIT's predictions and indirectly illuminate aspects of higher-dimensional supergravity at accessible energies.