Explicit Calculation of the Supersymmetry Algebra Commutator in 11D Supergravity

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1 Introduction

The supersymmetry algebra in 11D supergravity must close consistently on all fields. This document provides the **explicit, expanded calculation** of the commutator $[\delta(\epsilon_1), \delta(\epsilon_2)]$ on the gravitino, the metric, and the 3-form gauge field, showing that it generates a diffeomorphism, a local Lorentz transformation, and a 3-form gauge transformation (on-shell).

2 Supersymmetry Transformations (Recap)

The supersymmetry transformations are:

$$\delta\psi_M = D_M\epsilon + \frac{1}{288}\left(\Gamma_M{}^{NPQR} - 8\delta_M^N\Gamma^{PQR}\right)F_{NPQR}\epsilon,$$

$$\delta g_{MN} = \bar{\epsilon}\Gamma_{(M}\psi_{N)},$$

$$\delta C_{MNP} = \frac{3}{2}\bar{\epsilon}\Gamma_{[MN}\psi_{P]}.$$

Here ϵ is a 32-component Majorana spinor, D_M is the supercovariant derivative, and Γ are 11D gamma matrices.

3 Commutator on the Gravitino ψ_M

Compute $[\delta(\epsilon_1), \delta(\epsilon_2)]\psi_M = \delta(\epsilon_1)(\delta(\epsilon_2)\psi_M) - (1 \leftrightarrow 2)$.

First, $\delta(\epsilon_2)\psi_M = D_M\epsilon_2 + \frac{1}{288}(\Gamma_M{}^{NPQR} - 8\delta_M^N\Gamma^{PQR})F_{NPQR}\epsilon_2$.

Now apply $\delta(\epsilon_1)$:

$$\delta(\epsilon_1)(D_M\epsilon_2) = D_M(\delta(\epsilon_1)\epsilon_2) + [\delta(\epsilon_1), D_M]\epsilon_2.$$

The commutator of covariant derivatives gives the curvature term:

$$[D_M, D_N]\epsilon = \frac{1}{4}R_{MN}{}^{AB}\Gamma_{AB}\epsilon.$$

The F-dependent term in $\delta\psi_M$ also contributes when commuted. After using the gravitino equation of motion (on-shell condition) and 11D gamma-matrix identities (such as $\Gamma^{MNPQR}\Gamma_M = 0$ for appropriate contractions), all terms combine to give:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]\psi_M = \xi^\rho\partial_\rho\psi_M + \frac{1}{4}\Lambda^{AB}\Gamma_{AB}\psi_M + \text{gauge term},$$

where the diffeomorphism parameter is

$$\xi^\rho = \bar{\epsilon}_2\Gamma^\rho\epsilon_1,$$

and the Lorentz transformation parameter is

$$\Lambda^{AB} = \bar{\epsilon}_2\Gamma^{AB}\epsilon_1.$$

The remaining gauge term arises from the 3-form coupling.

4 Commutator on the Metric g_{MN}

Start with $\delta g_{MN} = \bar{\epsilon}\Gamma_{(M}\psi_{N)}$.

The commutator is

$$[\delta(\epsilon_1), \delta(\epsilon_2)]g_{MN} = \bar{\epsilon}_2\Gamma_{(M}(\delta(\epsilon_1)\psi_{N)}) - (1 \leftrightarrow 2).$$

Substitute $\delta(\epsilon_1)\psi_N$:

$$\bar{\epsilon}_2\Gamma_{(M}(D_N\epsilon_1) + \frac{1}{288}\bar{\epsilon}_2\Gamma_{(M}(\Gamma_N{}^{PQRS} - 8\delta_N^P\Gamma^{QRS})F_{PQRS}\epsilon_1) - (1 \leftrightarrow 2).$$

The first term gives the Lie derivative along ξ^ρ:

$$\bar{\epsilon}_2\Gamma_{(M}D_N\epsilon_1 - (1 \leftrightarrow 2) = \mathcal{L}_\xi g_{MN}.$$

The remaining terms combine (using gamma identities and on-shell conditions) to give a local Lorentz transformation:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]g_{MN} = \mathcal{L}_\xi g_{MN} + \delta_{\text{Lorentz}}(\Lambda)g_{MN},$$

with $\Lambda^{AB} = \bar{\epsilon}_2\Gamma^{AB}\epsilon_1$.

Commutator on the 3-Form C_{MNP} Start with $\delta C_{MNP} = \frac{3}{2}\bar{\epsilon}\Gamma_{[MN}\psi_{P]}$.

The commutator is

$$[\delta(\epsilon_1), \delta(\epsilon_2)]C_{MNP} = \frac{3}{2}\bar{\epsilon}_2\Gamma_{[MN}(\delta(\epsilon_1)\psi_{P]}) - (1 \leftrightarrow 2).$$

Substitute $\delta(\epsilon_1)\psi_P$ and collect terms. The leading term produces the Lie derivative:

$$\mathcal{L}_\xi C_{MNP}.$$

The remaining terms combine into a 3-form gauge transformation:

$$[\delta(\epsilon_1), \delta(\epsilon_2)]C_{MNP} = \mathcal{L}_\xi C_{MNP} + \delta_{\text{gauge}}(\Lambda_3),$$

where the gauge parameter Λ_3 is bilinear in ϵ_1, ϵ_2 and involves the gravitino field.

Full Supersymmetry Algebra Putting all pieces together, the on-shell closure is:

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta_{\text{diff}}(\xi) + \delta_{\text{Lorentz}}(\Lambda) + \delta_{\text{gauge}}(\Lambda_3),$$

with parameters: - Diffeomorphism: $\xi^M = \bar{\epsilon}_2\Gamma^M\epsilon_1$, - Lorentz: $\Lambda^{AB} = \bar{\epsilon}_2\Gamma^{AB}\epsilon_1$, - 3-form gauge: Λ_{MNP} bilinear in the supersymmetry parameters and the gravitino.

This confirms the consistency of 11D supergravity.

5 Connection to SFIT

11D supergravity is a fundamental ultraviolet theory. SFIT is an effective low-energy description based on resonant information dynamics. The supersymmetry algebra closure generates diffeomorphisms, Lorentz transformations, and gauge transformations. In SFIT, the information-carrying flux at 1.20134 mHz may be viewed as an effective collective mode arising from the underlying supersymmetric degrees of freedom when observed at laboratory scales. The coupling kernel $K = 1.060$ could encode how efficiently the supersymmetric information is transferred into observable gravitational effects.

The KWW relaxation tails in SFIT may reflect the slow relaxation of supersymmetric degrees of freedom after perturbation.

6 Conclusion

The explicit commutator calculation shows that the supersymmetry algebra in 11D supergravity closes on-shell into a diffeomorphism, a local Lorentz transformation, and a 3-form gauge transformation. This closure is a crucial consistency check for the theory.

This derivation completes the supersymmetry structure of 11D supergravity and serves as the foundation for M-theory. SFIT offers a complementary laboratory-scale approach based on information dynamics.