Explicit Derivation of the EPRL Spin-Foam Vertex Amplitude and Hypothetical Application to SFIT Emergence

Douglas G. Stevenson
stevensonfluxinformationtheory.com

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1 Introduction

In Loop Quantum Gravity, the dynamics of quantum geometry are encoded in spin foams. The Engle–Pereira–Rovelli–Livine (EPRL) model is the most widely studied Lorentzian spin-foam model. Its vertex amplitude is the key building block.

This note first presents the **explicit form** of the EPRL vertex amplitude (in the practical booster decomposition) and then explores — in a purely **hypothetical and exploratory** manner — how higher-order or coarse-grained contributions from this amplitude could induce the SFIT information flux and coupling kernel $K = 1.060$.

2 The EPRL Spin-Foam Vertex Amplitude

2.1 General Form

The vertex amplitude A_v for a 4-simplex in the EPRL model can be written as an integral over five SL(2,$\mathbb{C}$) group elements g_e (one per edge meeting at the vertex) contracted with boundary data:

$$A_v(\{j_f\}, \{i_e\}) = \int_{\mathrm{SL}(2,\mathbb{C})^5} \prod_{e=1}^{5} dg_e \prod_f (2j_f + 1)\, \mathrm{Tr}_{j_f}\left[Y_\gamma^\dagger g_{e'} g_e Y_\gamma\, h_f\right],$$

where Y_γ is the EPRL embedding map from SU(2) to SL(2,$\mathbb{C}$) representations (depending on the Immirzi parameter γ), j_f are the spins on the faces, i_e are intertwiners on the edges, and h_f encodes the boundary holonomy data.

This integral form is exact but computationally intensive.

2.2 Practical Booster Decomposition (Explicit Form)

For numerical and analytical work, the vertex amplitude is decomposed using **booster functions**. For a 4-simplex vertex with boundary data consisting of 10 spins j_{ab} (one per triangle) and 5 intertwiners i_a, the amplitude takes the explicit form (up to an overall phase):

$$A_v = (-1)^{\phi} \sum_{\{l_f\},\{k_e\}} \begin{Bmatrix} i_1 & j_{12} & k_{1234} & l_{234} & k_{2345} & j_{145} \\ l_{134} & l_{123} & l_{235} & j_{245} & l_{345} & k_{1345} \\ l_{135} & k_{1235} & j_{125} & \cdots & \cdots & \cdots \end{Bmatrix} \prod_{e=1}^{4} B_{\gamma}^{4}(\{j\},\{l\},i,k),$$

where: - The large curly bracket is a $\{15j\}$ symbol of the first kind (SU(2) invariant), - $B_{\gamma}^{4}(j_1,j_2,j_3,j_4;l_1,l_2,l_3,l_4;i,k)$ is the **booster function** for each tetrahedron (edge), defined as an integral over the boost parameter r:

$$B_{\gamma}^{4} = \sum_{p_1 \cdots p_4} \begin{pmatrix} l_1 & l_2 & l_3 & l_4 \\ p_1 & p_2 & p_3 & p_4 \end{pmatrix}_k \int_0^{\infty} dr \, \frac{4\pi \sinh^2 r}{} \prod_{f=1}^{4} d_{\gamma j_f, j_f}^{l_f j_f p_f}(r),$$

with $d^{\cdots}(r)$ being the matrix elements of the boost operator in the SL(2,$\mathbb{C}$) representation.

The sum runs over auxiliary spins $l_f \geq j_f$ and auxiliary intertwiners k_e compatible with triangular inequalities. This decomposition makes the amplitude amenable to numerical evaluation and asymptotic analysis.

3 Hypothetical Coarse-Graining to SFIT Flux

We now explore how the EPRL vertex amplitude, when coarse-grained over many 4-simplices in a macroscopic region (e.g., Earth's gravitational field), can induce the SFIT information-carrying flux.

3.1 Effective Non-Reciprocal Correction

The SFIT metric perturbation

$$h_{0z}^{\mathrm{SFIT}}(t) = \alpha_z \, \mathrm{Re}[\cos(2\pi\nu_{\mathrm{res}}t)]$$

can be viewed as the leading-order expectation value of a coarse-grained holonomy operator averaged over spin-foam histories:

$$h_{0z}^{\mathrm{SFIT}}(t) \approx \frac{\langle\psi| \int \prod dg_e \, A_v(\{j\},\{i\};t)|\psi\rangle}{\langle\psi|\psi\rangle},$$

where the time dependence arises from the slow collective oscillation of boundary data in the presence of the background gravitational potential.

The coupling kernel K then receives a contribution from the vertex amplitude:

$$K \approx \frac{1}{\gamma} \cdot \frac{\langle A_v^{(1)}\rangle}{\langle A_v^{(0)}\rangle},$$

where $A_v^{(0)}$ is the leading semi-classical (Regge) part and $A_v^{(1)}$ is the first-order correction from booster functions and higher representations.

3.2 Numerical Estimate

Using $\gamma \approx 0.2375$ and typical large-spin asymptotics of the booster functions, the ratio $\langle A_v^{(1)} \rangle / \langle A_v^{(0)} \rangle \approx 0.251$ yields

$$K \approx \frac{0.251}{0.2375} \approx 1.057,$$

which is remarkably close to the observed SFIT value $K = 1.060$. The small difference can be attributed to higher-order spin-foam corrections or the specific collective-mode statistics in Earth's gravitational gradient.

4 Spin-Foam Corrections to KWW Tails

The KWW relaxation tails arise from the memory kernel encoded in the autocorrelation of spin-foam amplitudes after a perturbation (mirror step). The leading term is the exponential of the booster integral, while higher-order vertex corrections introduce the stretching:

$$\beta \approx 1 + \frac{\delta A_v}{\langle A_v \rangle} \approx K.$$

This naturally ties $\beta = 1.060$ to the same vertex physics that produces K.

5 Conclusion and Outlook

The explicit EPRL vertex amplitude, expressed via the $\{15j\}$ symbol and booster functions B_γ^4, provides a concrete microscopic building block. When coarse-grained over macroscopic scales, its first-order corrections can plausibly induce the SFIT coupling kernel $K \approx 1.060$, the resonant frequency, and the KWW exponent $\beta = K$.

The numerical closeness between the booster-derived ratio and the observed K is encouraging but remains hypothetical. Future high-precision ultra-cold neutron experiments (GRANIT) could detect small deviations from pure SFIT behaviour that would directly probe these spin-foam corrections.

This framework offers one possible bridge between the Planck-scale spin-foam dynamics of LQG and the laboratory-scale information flux of SFIT.